Answer Sheet to the Written Exam

Corporate Finance and Incentives

December 2017

In order to achieve the maximal grade 12 for the course, the student must excel in all four problems.

The four problems jointly seek to test fulfillment of the course's learning outcomes: "After completing the course, the student should be able to:

Knowledge:

1. Understand, account for, define and identify the main methodologies, concepts and topics in Finance
2. Solve standard problems in Finance, partly using Excel
3. Criticize and discuss the main models in Finance, relating them to current issues in financial markets and corporate finance

Skills:

1. Manage the main topics and models in Finance
2. Organize material and analyze given problems, assessing standard models and results
3. Argue about financial topics, putting results into perspective, drawing on the relevant knowledge of the field

Competencies:

1. Bring into play the achieved knowledge and skills on new formal problems, and on given descriptions of situations in financial markets or corporations
2. Be prepared for more advanced models and topics in Finance."

Problems 1–3 are particularly focused on knowledge points 1 and 2, skills of type 1 and 2, competencies 1 and 2. Problem 4 emphasises knowledge points 1 and 3, skills 1 and 3, and competency 1.

Some numerical calculations may differ slightly depending on the software used for computation, so a little slack is allowed when grading the answers.

Problem 1 (APT 25%)

1) Define matrix

$$C = \begin{bmatrix} 7 & 3 & 1 \\ 15 & -2 & 1 \\ -5 & 8 & 1 \end{bmatrix}.$$

We seek the portfolio x that solves $x^T C = (0,0,1)$. Thus, $x^T = (0,0,1)\, C^{-1}$. Use matrix inversion in Excel to find $x^T = (-5.5, 3.55, 2.95)$.

2) The portfolio from 1) is risk-free and has expected return $(0.149, 0.173, 0.073)\, x = 0.01$, so the risk-free rate should be 1%.

3) The first pure factor portfolio is, transposed, $(1,0,1)\, C^{-1} = (-5, 3.3, 2.7)$. The second pure factor portfolio is, transposed, $(0,1,1)\, C^{-1} = (-4.5, 2.95, 2.55)$.

4) The first pure factor portfolio's expected return is $(0.149, 0.173, 0.073)\,(-5, 3.3, 2.7)^T = 0.023$, or 2.3%. Its risk premium is $2.3\% - 1\% = 1.3\%$. The expected return of the second pure factor portfolio is $(0.149, 0.173, 0.073)\,(-4.5, 2.95, 2.55)^T = 0.026$; risk premium 1.6%.

Problem 2 (Debt and Taxes 25%)

1) After deduction of the initial investment, the firm has 40 in the good state. Without any deduction of interest rates, it would pay corporate tax 30% of 40 which is 12. Once it also deducts interest rates, it cannot pay more corporate tax than 12.

2) In the good state, the firm receives 140. After paying corporate tax, it has at least 128 (according to point 1). When $P < 128$ it is therefore going to pay P to creditors. The given formula for D is then that of risk-neutral pricing.

3) Case $P = 50$. From 2), $D = 45.15$. The interest is $P - D = 4.85$. The tax bill is $.3\,(40 - 4.85) = 10.54$. In the good state, this leaves $140 - 50 - 10.54 = 79.46$ for owners. They receive 0 in the bad state. The present value of equity is thus $E = 0.65\,(79.46)\,/1.03 = 50.14$.

In case $P = 80$, we likewise find $D = 64.08$, interest 15.92, tax bill 3.90, $E = 33.31$.

In case $P = 110$, we find $D = 83.01$, interest 26.99, tax bill 7.22, equity value $E = 16.47$.

4) The value of the firm is $D + E$. When $P = 50$ this becomes 95.29. When $P = 80$ it is 97.38. When $P = 110$ it is 99.48. It is increasing with debt due to the tax shield of debt.

Problem 3 (Option Pricing 25%)

1) At time 0, we need to compute the probability p such that

$$10 = \frac{p18 + (1-p)\,6}{1.025},$$

solved by $p = 35.4\%$. With the corresponding method, we find at time 1 at the higher node that the probability of the up-branch in the tree is is 52.5%. Finally, at time 1 at the lower node, the probability of the up-branch is 30.7%.

2) At time 2, the values from top to bottom are dollars $(0, 1, 0, 6)$. At time 1 at the upper node, if the option is not exercised, its value is

$$\frac{52.5\%\$0 + 47.5\%\$1}{1.025} = \$0.46.$$

It is out of the money, so exercising gives \$0. The value is thus \$0.46. At the lower node, the continuation value is likewise \$4.06. Exercising at time 1 would give less, \$4. Its value is thus \$4.06. At time 0, the value is $P_0 = \$2.72$ (better than exercising for \$0).

3) At time 2, the values from top to bottom are dollars $(0, 2, 0, 7)$. At time 1 at the upper node, the value is thus \$0.93 (better than exercising for \$0). At the lower node the continuation value is \$4.73, but exercise at time 1 gives \$5, so in this case it is optimal to exercise at this node for value \$5. At time 0 the value is $P_0 = \$3.47$ (better than exercising for \$1).

4) At time 2, the values from top to bottom are dollars $(0, 3, 1, 8)$. At time 1 at the upper node, the value is \$1.39 (better than exercising for \$0). At the lower node the continuation value is \$5.71, but exercise gives the correct value \$6. At time 0 the value is $P_0 = \$4.26$ (better than exercising for \$2).

Problem 4 (Various Themes 25%)

1) This is the main equation from the CAPM, discussed in chapters 10–13 of Berk and DeMarzo. Risky assets are expected to earn a return premium over the risk-free rate, explained by systematic risk, here captured by the market portfolio of risky assets. Coefficient β_i captures the sensitivity of asset i to the market.

2) Chapter 17 in Berk and DeMarzo illustrates the agency issues in corporate finance. Chapter 29 discusses corporate governance remedies. New investors can be reluctant to provide financing to Uber if they do not trust that the firm will act in their interest. Shifting power to new investors can be the solution to an agency problem. Current owners recently hired the CEO, so he is likely to be carrying out plans to best protect their value, but the task is complicated since these owners are already in internal conflict. Travis Kalanick has already lost some power, and appears to lose some further power (super-voting rights) while gaining some other rights (chance to become CEO, dropped law-suit).

3) The WACC is shown as equation (12.12) by Berk and DeMarzo. Weights are the relative shares of debt and equity. To the extent that returns to creditors can be deducted from corporate taxation, the firm only experiences required return $r_D(1 - \tau_C)$ on debt. This point is further discussed in connection with the tax shield of debt in chapter 15, that copies the WACC definition in (15.5), and provides an illustration in figure 15.2. Chapter 12 further discusses the extent to which the WACC is a useful benchmark return on the firm's investments.